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A STUDY ON INFILTRATION AND RUNOFF ON A NATURAL FORESTED SLOPE

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A STUDY ON INFILTRATION AND RUNOFF ON A NATURAL FORESTED SLOPE

By

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Abstract

The relation between rainfall and runoff was investigated on a natural forested hillside from 1960 to 1965 with the use of a very small experimental catchment, which from the top downward has a profile of litter layer, humus layer, and sandy clay layer, and covered by pines, Japanese cypresses, and other smaller latifoliate trees. It was found that the litter layer creates runoff in the form of a flow of filmy water which runs downhill along the surface of litters, and the quantity of which is proportional to the rainfall intensity until it reaches the limit of $15\mu/\text{min}$. About 91% of the rainfall infiltrates from the humus layer down to the sandy clay layer, and the remaining 9% becomes a subsurface flow through the litter layer and the humus layer. It was estimated that the infiltration rate is proportional to the detention above the interface between the humus layer and the sandy clay layer.

1. Introduction

The problem of the repartition of rainwater into infiltration and runoff on the ground surface has been investigated intensively since Horton's concept of infiltration capacity, and many researchers have made many contributions with the use of different methods (Wisler and Brater [1964], Musgrave and Holtan [1964]) such as the infiltrometer, the lysimeter, the experimental catchment and so on.

Recent study on subsurface flow from the several upper soil layers (Whipkey 1965)) has shown the importance of the investigations on the behavior of rainwater within the upper soil layer of forested slopes.

This paper describes the results of an investigation on rainfall and runoff undertaken in a very small experimental catchment area. Special cautions were taken to keep the catchment under natural conditions as far as possible. Rainfall intensity and the runoff from each upper layer of the catchment was observed. Auxiliary data were given by measuring the temperature and soil moisture at various depths in the soil. The infiltration capacity of the catchment was so great that the runoff did not occur in the form of overland flow but in the form

of subsurface flow. Then the mechanism of the latter on natural forested mountainside was investigated along with the other related phenomena.

2. Methods

The experimental catchment is located on a slope about 40 m westward from the top of a hill in the Kamigamo Geophysical Observatory. This hill has a height of 180 m and is located in the line of hills forming the northern fringe of the Kyoto basin. The slope is covered with a mixture of Japanese cypresses, pines, bushes, and ferns. The soil profile is composed of the uppermost litter layer, a middle humus layer, and a sandy clay layer containing some pebbles. The upper litter layer has a coarse texture composed of the twigs and leaves of pines and latifoliate trees, and its thickness varies from zero to 2 cm. The lower litter layer is composed of a dense mixture of fractured leaves of pine and Japanese



Photo. 1 A view of the experimental catchment from the lower side.

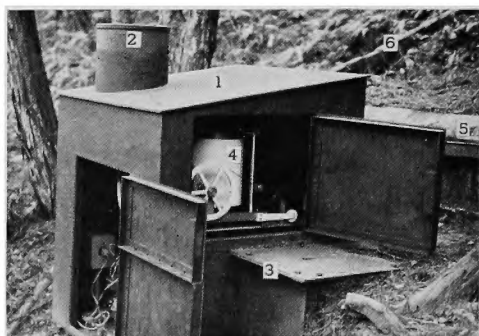


Photo. 2 A side view of the recording apparatus. 1-the cover, 2-the receiver of the rainfall intensity meter, 3-runoff water tank, 4-water level gage for runoff and rainfall intensity, 5-collector trough for runoff water, 6-the boundary bricks.

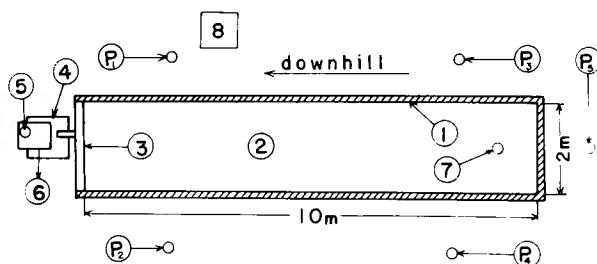


Fig. 1. A plan of the experimental catchment. 1-the boundary bricks, 2-catchment area, 3-runoff water collector, 4-runoff water tank, 5-the receiver of the rainfall intensity meter, 6-the cabinet of water level recorder, 7-the plot for temperature measurement, 8-the plot for soil moisture measurement, P₁-P₅-non-recording rain gages.

cypresses and has a relatively uniform thickness of about 0.5 cm. The humus layer is spongy and woven with roots of plants. Its average thickness is 8 cm. The experimental catchment which is bounded by a line of bricks jacked into the soil is 10 m long (along the direction of the slope) and 2 m wide, and has an inclination of 27° toward the west. A general situation of the catchment area is shown in Photos. 1 and 2, and in Fig. 1.

A Japanese Weather Bureau standard recording rain gage P_A^1 has been set on the horizontal plane at the top of the hill so as to be free from the interception by trees. The rainfall intensity meter P_P^1 which was specially designed for this study has a horizontal rainwater receiver with a diameter of 22.6 cm and sends the rainwater to the tank in the cabinet (Photo. 2). The water level of the tank is recorded on the same recording drum of the water level gage for runoff water. The rainfall intensity is obtained through the numerical differentiation of this record. Standard non-recording rain gages P_1 - P_5^1 has been installed perpendicular to the surface of the slope.

Runoff water from the catchment area is collected by the collector trough and introduced to the runoff water tank. The water level of the tank is recorded on the recording drum which is made to revolve by a synchronous motor. The collector trough was set at various position in order to collect the runoff water from the upper litter layer, the lower litter layer, or the humus layer. The observation of rainfall and runoff was carried out during each rainy season (usually from May to September) of the years from 1960 to 1965.

The temperature distribution was measured in 1964 by thermistors at four depths in the soil and in the air (at the height of 1 m). The distribution of soil moisture was measured in 1965 by five glass block moisture gages and a nylon soil moisture gage both having electrodes within them. Both the temperature and the soil moisture were recorded by an automatic recorder.

3. Distribution and losses of rainfall

In order to relate the runoff from a natural forested catchment to the rainfall intensity it is desirable to measure the intensity of the rainfall at the ground surface of the catchment area. But in this case rainfall intensity was measured at the top of the runoff water tank and the receiving plane was horizontal not parallel to the surface of the catchment area. So the comparative rain gages were installed at the top of the hill and around the catchment. The non-recording rain gages P_1 - P_5 and the rainfall intensity meter P_P give the throughfall.

1 In this paper each rain gage symbol also represents the rainfall intensity measured by the gage. If prime is added it represents the depth of rainfall.

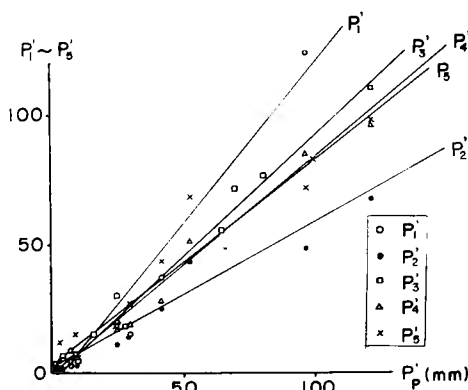


Fig. 2. Correlation between the values of the depth of rainfall measured by P_P and by P_1-P_5 .

graph it is seen that the throughfall onto the catchment is about 86% of that onto P_P if the former is assumed to be equal to the average of P_1-P_5 . Assuming that the stemflow is 19% of the throughfall, the intensity of net rainfall on the catchment (P) becomes:

$$\begin{aligned} P &= P_P \times 0.86 \times (1 + 0.19) \\ &= 1.02 P_P \end{aligned} \quad (1)$$

In this paper the rainfall intensity is expressed by P_P instead of the intensity of net rainfall calculated by eq. (1) because the latter is proportional to the former.

4. Intrerrelationship of rainfall intensity and runoff from each layer

During the period of the investigation from 1960 to 1965 the rainfall intensity observed was up to $2,000 \mu/\text{min}$, and no surface flow over the litter layer was

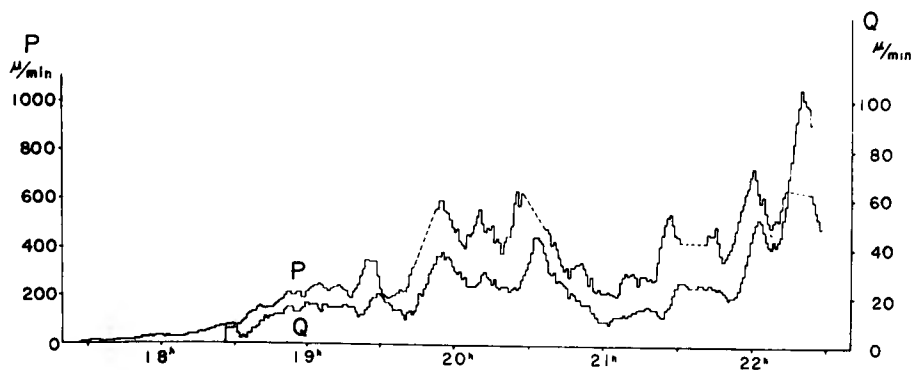


Fig. 3. Hydrograph of the runoff from the upper litter layer (Q) and rainfall intensity measured on Aug. 29, 1960.

observed. Thus the runoff observed during this period seems to belong to subsurface flow. Runoff from the upper litter layer, from the entire litter layer, and from both the litter layer and the humus layer was observed in 1960, 1961, and from 1962 to 1965, respectively. Representative hydrographs from each horizon (Q) are shown in Fig. 3 to 6 along with the rainfall intensity (P) measured by

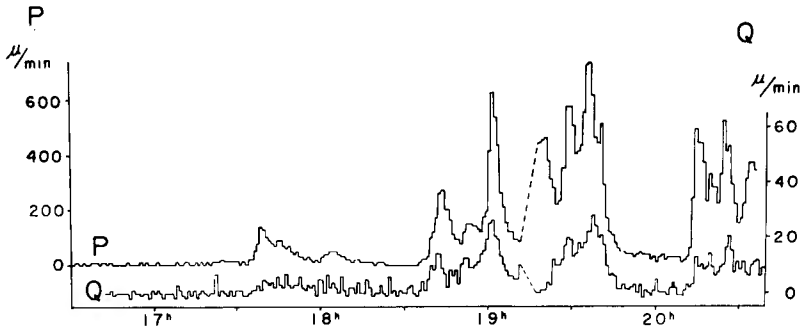


Fig. 4. Hydrograph of the runoff from the entire litter layer (Q) and the rainfall intensity (P) measured on June 9, 1961.

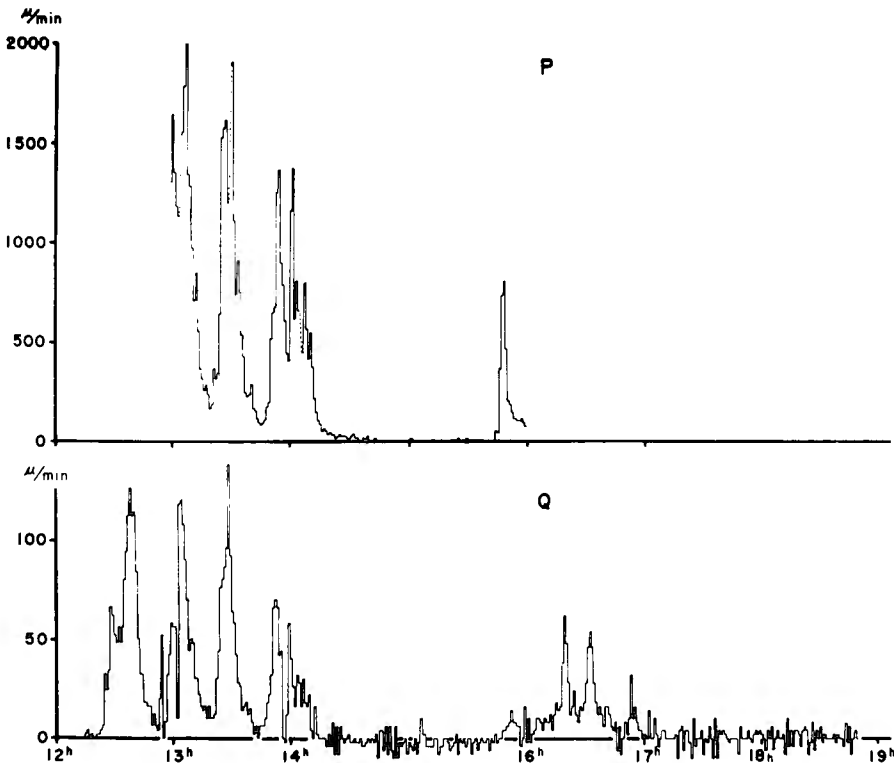


Fig. 5. Hydrograph of the runoff from both the litter layer and the humus layer (Q) and the rainfall intensity (P) measured on Sept. 17, 1965.

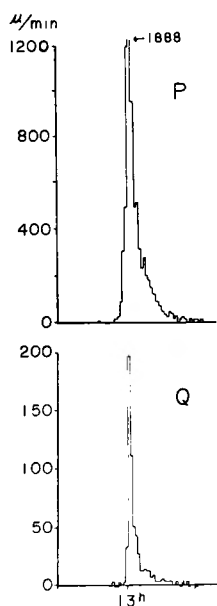


Fig. 6. Hydrograph of the runoff from both the litter layer and the humus layer (Q) and the rainfall intensity (P) measured on Aug. 3, 1965 when rainfall intensity was very high.

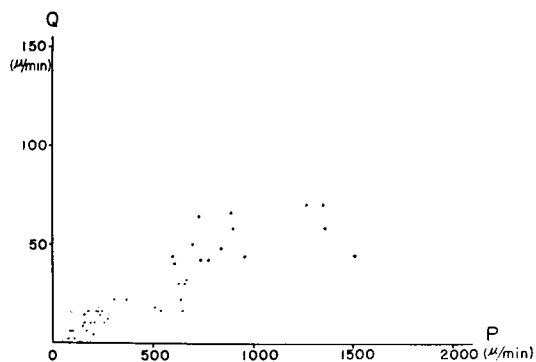


Fig. 9. Correlation between P and Q of Fig. 5.

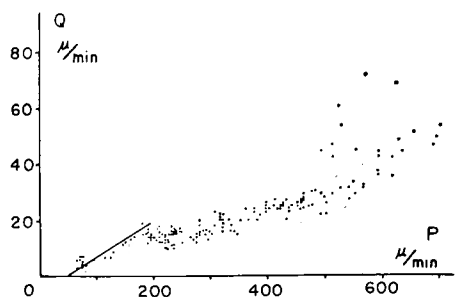


Fig. 7. Correlation between P and Q of Fig. 3.

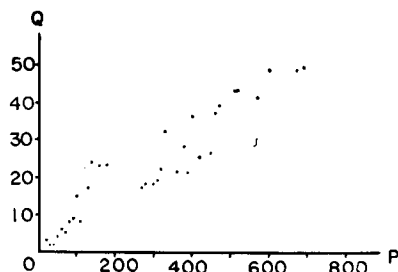


Fig. 8. Correlation between P and Q of Fig. 4.

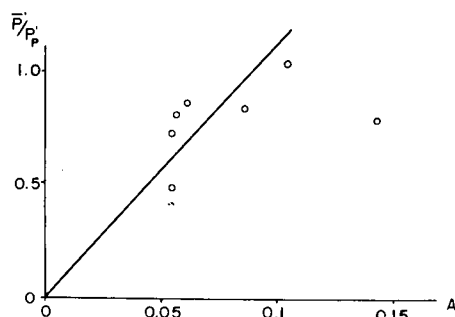


Fig. 10. Correlation between the parameter A in Table 3 and the ratio \bar{P}/P' .

P_P . It is remarkable that the curves of rainfall intensity and of runoff intensity resemble each other in every graph. The same relation holds when rainfall is of large duration (Fig. 3) or short duration and high intensity (Fig. 6). The time lag between rainfall and runoff is usually about two minutes. So the smoothing effect is insignificant.

Correlations between rainfall intensity and runoff intensity are given in Figs.

7 to 9 corresponding to the hydrographs shown in Figs. 3 to 5. The relation between rainfall intensity and runoff from the litter layer (Fig. 7 and 8) is nearly linear, but has two bends at $P=200$ and $300\mu/\text{min}$. It is noted that runoff (Q) from the litter layer is nearly constant when P is between 200 and $300\mu/\text{min}$. The relation between rainfall intensity and runoff from both the litter layer and the humus layer (Fig. 9) is fairly linear.

The results of the regression analysis of the P - Q correlation are shown in Table 1 to 3. It can be noted in these tables that the parameters of regression line for each storm disperse extremely though within any single storm dispersion from the regression line is relatively small (cf. Figs. 7 to 9). One of the possible causes is the effect of wind. There exist some uncertainties in the relation between the depths of rainfall measured by rainfall intensity meter P_P and by non-recording rain gages P_1 - P_s (Fig. 3). Fig. 10 shows that the ratio of \bar{P}'

Table 1. Parameters of regression curves of P - Q correlation for the runoff from the upper litter layer observed in 1960

Equation of the regression curve		$Q=AP+B$ ($P>200\mu/\text{min}$)		$Q=CP^n$ ($P>300\mu/\text{min}$)	
No. of storms	date	A	B	n	C
1	July 8	0.039	+0.24	1.19	0.018
2	Aug. 5	0.086	+3.97		
3	Aug. 11	0.046	-0.35		
4	Aug. 12-13	0.046	-5.09		
5	Aug. 29-30	0.129	-6.10		
	mean	0.069	-1.56		

Table 2. Parameters of regression curve of P - Q correlation for the runoff from the entire litter layer observed in 1961

Equation of the regression curve		$Q=AP+B$ ($P<200\mu/\text{min}$)		$Q=CP^n$ ($P>300\mu/\text{min}$)	
No. of storms	date	A	B	n	C
1	May 3-4	0.046	+0.85	1.20	0.021
2	June 9	0.137	-1.73		
3	June 17	0.060	+1.88	1.15	0.019
4	June 28-29	0.032	+3.48		
5	June 29	0.024	+1.68	1.51	0.002
6	July 6-7	0.093	-1.42	1.63	0.0008
7	July 11	0.059	-1.85		
8	Aug. 5	0.072	-1.49	1.45	0.004
9	Aug. 9	0.037	+0.58		
10	Sept. 14-15	0.177	+2.84		
	mean	0.0737	+0.52		

(the average depth by P_1-P_5) to $P_{P'}$ is closely correlated to the value of the parameter A. Assuming that \bar{P}' represents the average rainfall onto the catchment, intensity on the ground surface of the catchment area (P_o) can be estimated from the following relation :

$$P_o = P_P \times P' / P_{P'} \quad (2)$$

This estimation is available only for the data of the years from 1963 to 1965. The values of A of the regression line re-calculated using P_o are shown in Table 3, which are somewhat less scattered than the value before re-calculation. Most of the residual scattering might be due to the complex distribution of wind speed over the catchment and each rain gage.

Table 3. Parameters of the regression lines of P-Q correlation for the runoff from both the litter layer and the humus layer

Equation of the regression line: $Q = AP + B$

From the observation in 1962

No. of storms	date	A	B
1	May 14-15	0.0355	+3.87
2	June 22	0.1333	-7.61
3	June 25	0.0653	+1.46
4	July 4	0.0478	+0.58
5	July 5	0.0717	-2.98
6	July 10	0.0691	+0.66
7a	July 27	0.2121	-1.32
7b	July 27	0.1484	+0.94
7c	July 27	0.1362	+6.07
7d	July 27	0.1248	+6.24
8a	Aug. 9	0.2499	+0.67
8b	Aug. 9	0.1628	+1.86
9	Aug. 13	0.2306	+3.50
10a	Sept. 4	0.1454	+0.29
10b	Sept. 4	0.1094	+1.61
11	Oct. 4	0.1641	+0.97

From the observation in 1963

No. of storms	date	A	B	A ^{1/2}
1a	June 6	0.0402	+5.01	} 0.0758
1b	June 6	0.0333	+1.70	
2	June 14	0.0707	+1.71	
3	July 12	0.0537	-6.08	
4	Aug. 10	0.0688	+1.07	
5	Aug. 17	0.0665	+2.68	
6	Aug. 30	0.0621	-0.06	
7	Aug. 31	0.0476	+1.60	

From the observation in 1964

No. of storms	date	A	B	A' ²
1	June 27-28	0.0521	+2.07	0.0728
2a	July 8-9	0.1089	-1.42	
2b	July 8-9	0.1208	+1.41	
3a	July 10-11	0.0454	+2.76	
3b	July 10-11	0.0677	+1.25	
4a	July 17-18	0.0485	+0.18	
4b	July 17-18	0.0515	+1.33	
5a	July 18-19	0.0530	+0.86	
5b	July 18-19	0.0956	-11.27	0.1032
6	Aug. 23	0.1432	-1.75	
7	Sept. 24	0.1047	+1.88	
8	Sept. 25	0.1617	+0.10	0.1025
9	Oct. ?	0.0855	+1.30	

From the observation in 1965

No. of storms	date	A	B	A' ²
1a	July 5-6	0.0655	-2.58	0.0710
1b	July 5-6	0.0568	-0.76	
1c	July 5-6	0.0487	-1.47	
2a	July 6-7	0.0391	-0.69	
2b	July 6-7	0.0255	+1.38	
3	Aug. 3	0.0839	-3.82	0.1022
4	Sept. 2	0.1256	+0.46	
5	Sept. 6	0.1191	+2.84	
6	Sept. 9	0.0754	+6.86	
7	Sept. 10	0.0952	+1.17	
8	Sept. 14	0.0937	+2.74	
9	Sept. 16	0.0504 ¹	—	
10a	Sept. 17	0.0443	+4.37	
10b	Sept. 17	0.0515	-0.20	
mean (1962-1965)		0.0916	+1.11	0.0879

1 The ratio of the depth of runoff to that of rainfall

2 The value of A modified by eq. (2)

5. Variation of temperature and soil moisture distribution within the soil profile during rainfall

The soil temperature was measured at four points within the soil profile along with air temperature at the height of 1 m in 1964. The objective of this measurement was to estimate the depth of the front of infiltrating rainwater during the rainfall. But the change of the soil temperature during the rainfall was insignificant. Fig. 11 shows the temperature distribution before, during, and after the

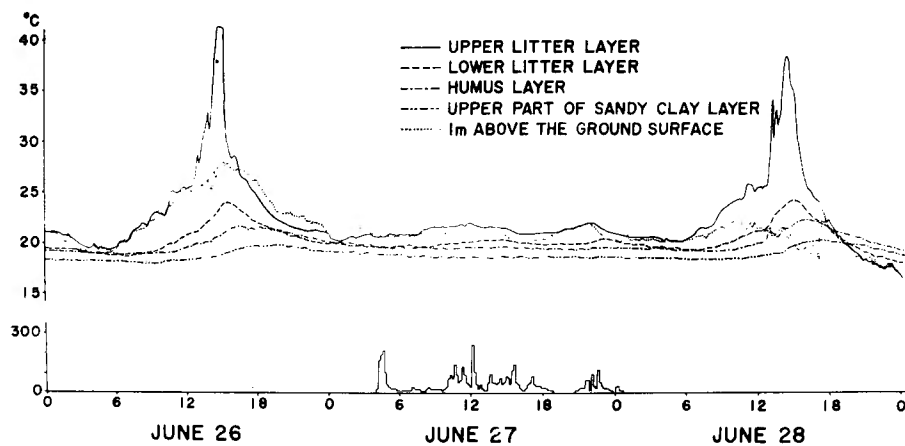


Fig. 11. The variation of the temperature distribution before, during, and after the rainfall along with rainfall intensity measured in 1964. The lower graph shows the rainfall intensity in μ/min .

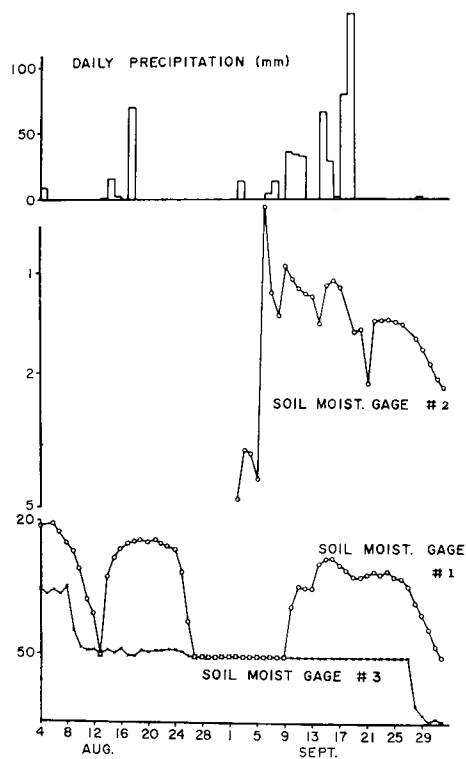


Fig. 12. A long term variation of the distribution of soil moisture measured in 1965 in terms of the impedance between the electrodes ($K\Omega$). #1-at the interface of the litter layer and the humus layer, #2-5cm below the surface of the sandy clay layer, #3-15cm below the surface of the sandy clay layer. The upper graph shows the daily rainfall by rain gage P_A .

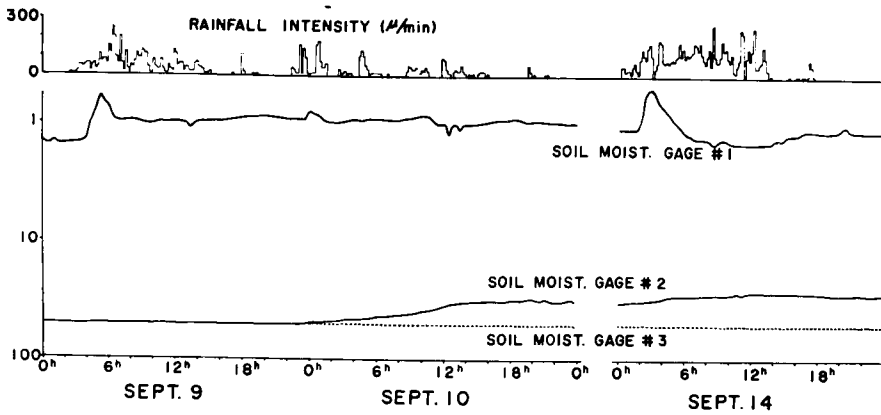


Fig. 13. Examples of short term variation of the distribution of soil moisture measured in 1965. The number of the soil moisture gages is the same with that of Fig. 12. The upper graph shows the variation of rainfall intensity by P_A .

rainfall at various depths with the intensity of rainfall. On June 26 th the temperature distribution showed the variation usual on fine days. On 27 th, when it was rainy almost thoroughout the day, the soil temperature did not rise. On the third day it was fine again and the normal temperature variation recurred. During the other rainfalls the situation was the same. It follows that the temperature variation within the soil profile is mainly caused by the heat conduction and radiation from and into the atmosphere and that the cooling effect of the soil by the infiltration of relatively cold rainwater is very weak.

The soil moisture was measured by a nylon moisture gage and glass block moisture gages at six depths, among which three showed no variation in the impedance between the electrodes because the capillary connection between the soil and the block was not good. In Fig. 12 a long term variation of soil moisture content is shown along with daily rainfall. It is seen that there is a rather good correlation between the long term rainfall and the soil moisture at each depth. Owing to the rainy days before the beginning of the measurement, the earlier part of the graph shows high soil moisture. Fig. 13 shows examples of short term variation in soil moisture. For such a short term variation, the correlation between soil moisture and rainfall is not very good, though there exists a certain correspondence between the rainfall intensity and the change of the soil moisture at the soil moisture gages #1 and #2.

6. Mechanism of infiltration and runoff of rainwater

Figs. 7 and 8 suggest that the mechanism of infiltration and runoff in the upper litter layer is the same with that in the entire litter layer. Though the upper litter layer does not create runoff when the rainfall intensity is lower

than $22.6\mu/\text{min}$ as an average while the lower litter layer creates the latter in the same time, when the rainfall intensity exceeds $22.6\mu/\text{min}$ the difference between runoff from the entire litter layer and that from the upper litter layer becomes insignificant. When rainfall intensity is less than $200\mu/\text{min}$, about 7 % of the rainwater becomes runoff water and the remaining 93 % infiltrates to the underlying humus layer. Considering that the permeability of the humus layer is far larger than the maximum rainfall intensity observed, it is concluded that the runoff from the litter layer consists of filmy water flowing downhill on the surface of litters. This flow is possible because the dimension of the litters is usually much larger in the direction parallel to the ground surface than in the direction perpendicular to it, and the filmy water on the surface of the litter inevitably flows in the direction parallel to the ground surface.

The thickness of the water film on the surface of litters is limited and consequently the quantity of such a runoff is also limited. This limit seems to correspond to the horizontal part of the P - Q regression curve of Figs. 7 and 8 when P is between 200 and $300\mu/\text{min}$.

The runoff from the humus layer is given by the difference between the runoff from the litter layer and that from both the litter layer and the humus layer. But it would be natural to treat the runoff from both the litter layer and the humus layer as a single component of runoff because the relation of rainfall and runoff from both layers is expressed as a single straight line, whereas the relation of rainfall and the runoff from the humus layer calculated is expressed as a succession of two straight lines and a curve. Thus the bends of P - Q curve for the litter layer can be interpreted as the result of the change of repartition of rainwater between the two layers.

The runoff from both layers is proportional to (equals 9 % of) the rainfall, therefore the infiltration is also proportional to the rainfall. A similar mechanism of runoff to that of the runoff from the litter layer can not be applied to the runoff from these layers because there exists no remarkable anisotropy in the texture of the humus layer which is the main cause of the occurrence of runoff within the litter layer.

The infiltration capacity of the catchment area was measured by a buffered infiltrometer with an inner tube of a diameter of 130.05 mm and an outer tube of a diameter of 254.0 mm. These two tubes were jacked to the depth of 10 cm into the soil profile and water was poured onto the two sections such a way that a third of the ground surface was always covered by surface water. The result of the infiltration capacity thus measured is given in Fig. 14. The measurement was made two times at interval of one day. It is noted that the variation of infiltration capacity is insignificant except for the initial stage. This

graph does not reflect the existence of multi-layering near the surface which is thought to be the result of the extraordinarily high permeability of the litter layer and the humus layer. The initial high rate infiltration seems to become absorbed in filling the pores in these two layers. Because of the high permeability of the upper two layers hydrostatic pressure nearly corresponding to the water column of the depth of the upper two layers is applied to the surface of the

sandy clay layer. Thus the ultimate infiltration capacity maintains a high level of $10,000\mu/\text{min}$ i. e. 600 mm/h . In the case of the infiltration of rainfall the water surface does not rise to the surface of the litter layer, perhaps staying within the humus layer or the litter layer. When the depth of water above the interface between the humus layer and the sandy clay layer is zero, infiltration capacity approaches the permeability of the sandy clay layer which is of the order of $100\mu/\text{min}$ according to a rough test. Then the infiltration capacity during the rainfall varies from 100 to $10,000\mu/\text{min}$ according to the depth of water above the interface.

Hayami [1956] and Yuhara [1964] have introduced theoretical solutions of the time change of the infiltration rate. Hayami's solution is as follows:

$$f = K \left(1 + \alpha \frac{H-b}{z} \right) \quad (3)$$

where f is infiltration capacity, K is permeability, α is porosity, z is the depth of the front of infiltration, b is the capillary potential at the front, and H is the depth of surface detention which in this case should be replaced by the height of the water surface above the interface between the humus layer and the sandy clay layer. The result of the measurement of soil moisture (Fig. 13) suggests that the rain water does not infiltrate to the depth of 15 cm below the surface of the sandy clay layer in a short time. If it is assumed that $\alpha H \gg z$ and $H \gg b$, the right side of eq. (3) approximates to $\alpha H/z$, and is consequently proportional to H . It is known that the velocity of the runoff water through the humus layer is constant (Takasao and Kishimoto [1961]), therefore runoff is also proportional to H . In this situation H is proportional to P (Okunishi [1963])

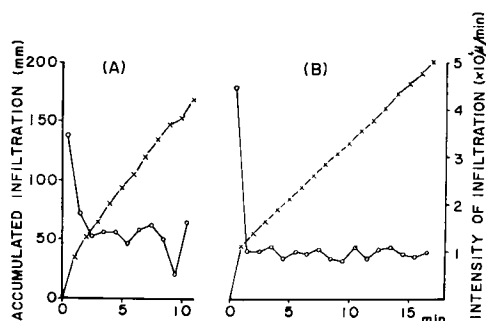


Fig. 14. The time change of the infiltration capacity measured by buffered infiltrometer on Oct. 22 (graph A) and on Oct. 23 (graph B), 1964. circle: infiltration capacity, cross: accumulated curve of infiltration capacity.

but when rainfall continues and also the depth of infiltration increases, the depth of the front of infiltration increases steadily if there is no significant sideways flow within the sandy clay layer, and the assumption that $\alpha H \gg z$ may not hold any more. In reality the proportionality between rainfall intensity and the infiltration rate holds even when the depth of rainfall becomes 100 mm or more.

7. Initial loss of rainwater

At the initial stage of rainfall it was often found that runoff does not occur until the depth of rainfall attains a certain value which is independent of rainfall intensity.

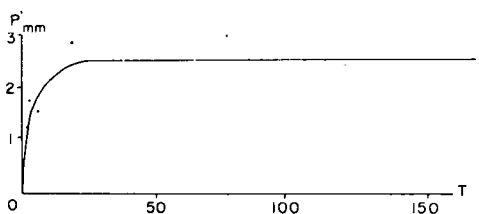


Fig. 15. Initial loss of rainfall (P') as a function of the time elapsed since the previous rainfall T (in hour).

In Fig. 15 is shown the threshold value of accumulated rainfall necessary to initiate runoff from the litter layer as a function of the time (T) that has elapsed since the previous rainfall. It is noted that this threshold value is far less than the depth of the initial infiltration of Fig. 14 used to

fill the pores of the litter layer and the humus layer, and already becomes constant when T is about 24 hours. This quantity is thought to correspond to the initial interception by ferns growing at a level lower than that of the receiver of rainfall intensity meter and to the initial detention on the surface of litters.

8. Conclusions

From the investigations of 1960 to 1965 at the experimental catchment of Kamigamo, the mechanism of the occurrence of runoff from the upper three layers of the catchment is estimated as follows ;

The litter layer and the humus layer have such a large infiltration capacity that pure surface runoff can not occur. The litter layer can transfer the filmy water sticking on each litter surface sideways along the surface of the catchment and thereby give a component of runoff whose quantity is proportional to the rainfall intensity. These flow has a limit in its quantity which is equivalent to the rainfall intensity of $15\mu/\text{min}$. When the rainfall intensity exceeds $300\mu/\text{min}$ the flow through the humus layer overflows to the litter layer and the flow through the litter layer increases again with the rainfall intensity.

Seepage of rainwater from the litter and the humus layers down to the sandy clay layer seems to be proportional to the detention above the surface of the sandy clay layer. Therefore infiltration, runoff, and detention all become

proportional to rainfall intensity.

The mechanism of the infiltration proportional to rainfall intensity was not fully elucidated. In order to solve this problem more precise observations of the flow of rainwater within each layer will be necessary.

Acknowledgment

The author is indebted to Dr. S. Hayami and Dr. Y. Fukuo for their kind guidance and advice throughout this investigation.

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